Creating Palindromes in Different Bases

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Abstract

Creating palindromes with 4 different methods and in negative bases are studied. The "No Carry" method is found to produce the best results without Lychrel candidates or Lychrel numbers and resulting in a palindrome in no more than 2 steps. All methods are shown to produce a variety of interesting results, which are analyzed and discussed. The "Subtract" process is found to be able to create unchangeable Lychrel numbers, such as 1012 in base 3. The "No Carry" method is discovered to result in uneven distribution of digits of the palindromes in even bases.

Introduction

After reading a book from the last QED, I had a lot of interesting ideas in mind. The most interesting idea was palindromes. Palindromes in Mathematics are numbers that can be flipped over (reversed) and still be the same. For example, 2112 stays the same when you flip it, but 2768 is not a palindrome, because when you flip it you will get 8672. I started researching about this topic and came across the method called the "reverse and add" process. The process works by taking a number, flipping it, adding to the original number, and doing it again and again until you get a palindrome. For example, let's take 24. 24 + 42 = 66 which is a palindrome. Not all numbers turn into palindromes. Those numbers are called Lychrel numbers. For example, 196 is the smallest Lychrel candidate in base 10. It's a Lychrel candidate, because it's not proven that it can't result in a palindrome. Although math enthusiasts went over billions of steps, but they can't get a palindrome from 196.

I was curious about creating palindromes in negative bases, because nobody did it and I never even heard about negative bases. Secondly, I wondered whether I could change the standard “Reverse and Add” process to make it better for producing palindromes.

Method

I used Swift coding language to study different methods to create palindromes. For each method my program has analyzed 1 million numbers. In some cases, I ran it for more than 10 millions numbers to find, for example, Lychrel numbers in the “No Last Carry” method. I studied bases from 2 to 25, including negative bases from -25 to -2. For each method I tried to find the 1st Lychrel number, Lychrel density, and Lychrel Delay. The latter means the largest number of steps it takes to produce a palindrome.

Standard (Reverse-and-Add)

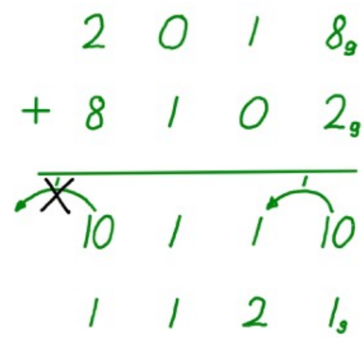
I used the standard “Reverse-and-Add” method to reproduce the known results and to make sure that my code works correctly. In additions, I noticed the following patterns:

$$\begin{array}{ccccc} & 2 & 0 & 1 & 8_9 \\ + & 8 & 1 & 0 & 2_9 \\ \hline & 10 & 1 & 1 & 10 \\ 1 & 1 & 1 & 2 & 1_9 \end{array}$$

- Digits in palindromes are not evenly distributed
- Palindromes cannot have zeros as the unit digit because the starting digit is the same as the units
- In unit digit distribution, 1 is used more often than other digits.

No Last Carry

I believed that the standard method produces Lychrel numbers because numbers grow with each step. In the “No Last Carry” the last carry is ignored as shown on the picture to the right. The main results are:

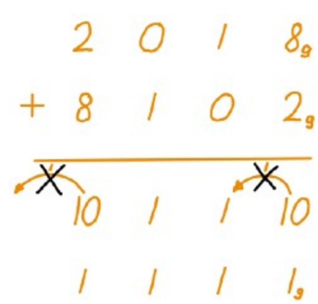


- Lychrel density is very close to 0%
- The 1st Lychrel numbers are larger than in any other rule.
- Lychrel density increases and then decreases.

Because numbers are limited by the number of digits of the original number, all Lychrel numbers repeat in a cycle.

No Carry

The “No Carry” method is based on the standard “Reversed and Add” method, but all carries are ignored as shown in the picture. Here are the main findings:



- The maximum number of steps is 2. Usually a palindrome is produced in one step but if the leading digits add up to 0 with a carry, it leads to the 2nd step.
- All digits except 0 are evenly distributed in the unit place
- 0 is less used in palindromes than other digits
- In even bases even digits are used more often than odd digits.
- In odd bases all digits except 0 are evenly distributed

Subtract

The “Reverse and Subtract” method finds the positive difference between the original number and reversed number. Here are the main findings.

$$
\begin{array}{rrrrr}
 & 8 & 1 & 0 & 2_9 \\
- & 2 & 0 & 1 & 8_9 \\
\hline
 & 6 & 1 & -1 & -6 \\
 & 6 & 0 & 7 & 3_9
\end{array}
$$

- In even bases the least and greatest digits are used the most.
- In odd bases the least, middle, and greatest digits are used the most.
- In even bases two middle numbers (4 and 5 for base 10) are used equally.
- No Lychrel numbers are found in base 2 and 4
- 1st Lychrel numbers are written the same in different bases (1012, 13, and 14).
- 1012 in base 3 is not changed by the procedure. Let’s call it an unchangeable Lychrel. Basically, the reversed number is twice as large as the original number and that is why the number stays unchanged.

Nega (Negative Base)

At the beginning of the project, I didn’t know that negative bases exist. The ideas of negative bases is simple but figuring out how add numbers in negative bases is a bit complicated as it may include both a carry and borrow.

$$
\begin{array}{rrrrrr}
 & & 2 & 0 & 1 & 8_{-9} \\
 & + & 8 & 1 & 0 & 2_{-9} \\
\hline
 & & 10 & 1 & 1 & 10 \\
1 & 8 & 1 & 1 & 0 & 1_{-9}
\end{array}
$$

- In even bases the biggest digit is used in palindromes more often than other digits
- In odd bases some of the odd digits are used more often than other digits.

- The density of Lychrel candidates can increase and decrease with increasing the number of digits in base 10 representation.

Conclusion

In this project I researched 4 different methods of creating palindromes and explored the standard “reversed and add” method in negative bases. It’s turned out that the best process to produce palindromes is the “no carry” method. It doesn’t result in Lychrel candidates or numbers and a palindrome can be obtained in no more than 2 steps. All methods produce a variety investing results. One of the most surprising results was the discovery of unchangeable Lychrel numbers in the “subtract” process, where the number stays always the same. Also, it was interesting to discover in the “no carry” method the reason why in even bases even digits are used more often than odd digits.

The most challenging part of this project was learning Swift coding language and using coding for a mathematical project. I worked on a computer-assisted research for the first time and I hope the new skills will help me in the future. Also I learned about negative bases that I didn’t even know existed.

There are a lot of directions for further research. What other Lynchrel numbers are unchangeable? What happens when we use 4 new methods but in negative bases? Why the distribution of digits is uneven in most cases?

Bibliography

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