Magic Perimeter Triangles

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by Khushi K. Pusnur

Grade 5, Chicago Free School

Abstract

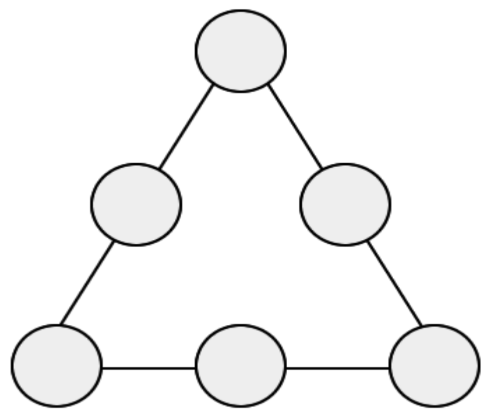
My project is about Magic Perimeter Triangles. I am going to solve different Magic triangles; show the different triangles; and present the different patterns. I am also going to show how different types of magic triangles share common patterns. The Magic Triangle problem is not like many other problems because it doesn't have a straightforward answer. This is why it is so interesting to solve.

Introduction

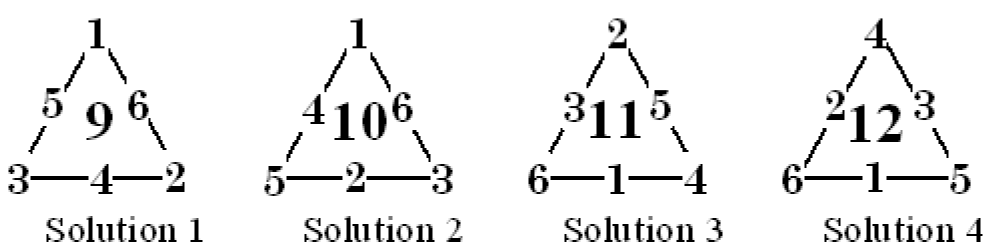
A Magic Triangle is like a triangle game. It is basically a triangle with numbers on the side, and one must find the sum of the numbers, and each side must have the same sum. I am going to solve the different magic triangles and explain the different patterns. First, I am going to describe an order three perimeter triangle. Then I am going to describe an order four parameter triangle.

Order three triangle

The first step in making an order three perimeter Magic Triangle is to draw a triangle with one bubble on each vertex and one on each side. Using numbers 1-6 one has to fill in all the bubbles, without repeating any of the numbers. The goal is to make each side equal the same sum.



The four solutions of an order three triangle are 9, 10, 11, and 12.[1]



Observations and pattern recognition:

1. The sum 9 is the smallest of these solutions. The vertices here include 1,2 and 3, which are also the smallest numbers in the series of numbers. Similarly, the biggest sum, 12, contains the vertices 4,5 and 6, which are likewise the biggest numbers in the series. ***So,***

[1] It might seem that there are more ways to write each solution, but it is the same triangle flipped in three different ways with a different side being the base each time.

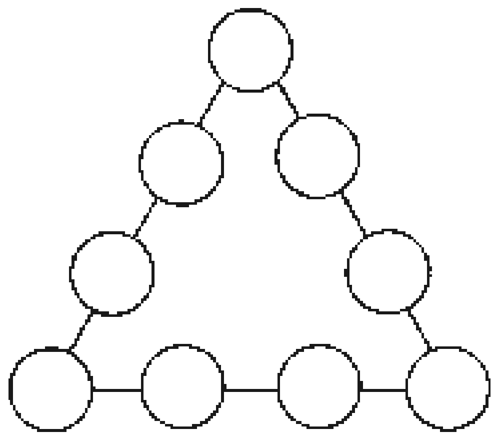
the smallest solution always has the smallest numbers in the vertices and the biggest solution has the biggest numbers in the vertices.

2. Each vertex has an opposite side, for example, in the case of solution 9, 3’s opposite side is 6, 1’s opposite side is 4 and 2's opposite side is 5. Notice that 6 - 3 = 3, 4 - 1 = 3 and 5 - 2 = 3. Three is the common difference between the vertex 4 and the number on the opposite side. The same pattern is true with the solution 10, 11, and 12. The common difference in solution ten’s triangle is 1; the common difference in solution elevens triangle is 1; and triangle twelve’s common difference is 3. ***So, in any possible solution, the difference between the vertex and the number on the opposite side is always the same.***
3. For the triangle with the solution 9, if one adds the vertex 1 plus the opposite side number 4 you get 5, which is an odd number. If one adds the vertex 2 with the opposite side number 5 you get 7, which is also odd. And if you add the vertex 3 and the side number 6 you get 9, which is odd too. All of these are odd numbers. ***So every vertex plus the opposite side number for any order three triangle will equal an odd number.***
4. In the case of triangle solutions 9, 10, and 11, there is another pattern. If you add the two numbers (5 and 4) for the triangle solution 9, you get a sum of 9. For triangle solution 10, if you add the numbers (6 and 4), you get 10. And for solution 11 if you add (6 and 5) you get 11. But for the case of solution 12, with the numbers 1-6, the only possible way to get twelve is 6+6 but you cannot repeat a number. ***It looks like in each solution, except 12, there are always two numbers on two different sides that will always add up to the solution.***

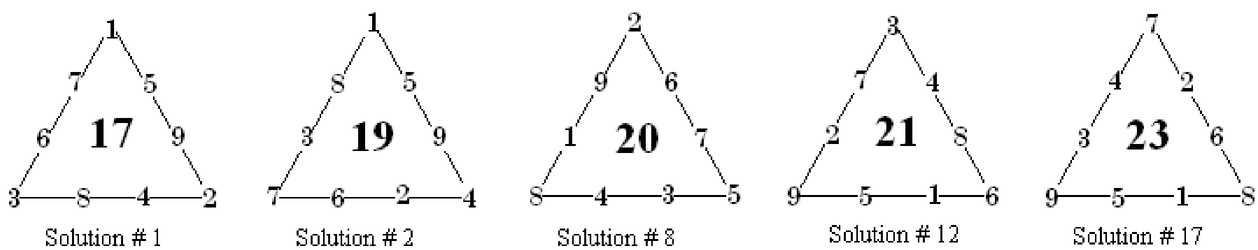
5. The vertices for solution 9 are 1,2,3, the vertices for solution 10 are 1,3,5, the vertices for solution 11 are 2,4,6, and the vertices for solution 12 are 4,5,6. At first glance, you may not see a pattern, but if you add up all the vertices for each number you get the numbers- 6, 9, 12 and 15 which are all consecutive multiples of 3. (There are also 3 sides on a triangle.) This same pattern is true for the side numbers. If you add up the numbers on the side for solution 9, 10, 11, and 12 you get 15, 12, 9, and 6! The same multiples of 3 but in reverse order. ***So, for order three-triangles, the sum of the vertex numbers of different solutions are always multiples of three. Likewise, the sum of the side numbers of the different solutions are also always multiples of three.***
6. The vertices for solution 10 are 1,3 and 5, which are all odd. The vertices for solution 11 are 2,4 and 6, which are all even. ***It seems, for order three triangles, except the smallest and the largest solutions, even solutions have odd vertices and odd solutions have even vertices.***

Order four triangle

A four order triangle is made a lot like a three order triangle but it has two bubbles on each side instead of just one. And instead of using the numbers 1-6 you must use the numbers 1-9.



There are 5 possible solutions to this triangle- 17, 19, 20, 21, and 23.[2]



Observation and pattern recognition:

1. Seventeen is the smallest solution and its vertices, 1, 2 and 3, are also the smallest of the number series 1-9. The largest solution 23 has the largest numbers 7, 8, and 9 in the vertices. ***So, like order three triangles, in order four triangles also, the smallest solution***

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[2] As in the three-order triangles, each solution can be flipped in three ways. Further, in this case, you can also switch the side numbers, thus, making a total of five ways to write each solution.

always has the smallest numbers in the vertices and the biggest solution has the biggest numbers in the vertices.

2. There is another pattern that is also similar to 3-order triangles. Each side has an opposite vertex, and if you add the two side numbers and subtract the opposite vertex you get the same common difference for every side and vertex of a particular triable. For solution 17, the common difference is 11; for solution 19 the difference is 7, for solution 20 it is 5, for 21 it is 3 and for 23 the difference is 1. All of these common differences are odd numbers. ***So, like in order three triangle, in any possible solution of a four order triangle the difference between the vertex and the numbers on the opposite side are always the same.***
3. For solution 17, the sum of the side numbers and the opposite vertex are 15, 17, and 13, all of which are odd numbers. For solution 19, the sum of the side numbers and the opposite vertex are 15, 21, and 9, which are again odd numbers. For solution 20, the sums are 15, 13, and 21; for solution 21, the sums are 15, 21, and 9; and for solution 23, the sums are 13, 15, and 17. All these sums are odd numbers. ***So, like order three triangles, in order four trriangles too every vertex plus the opposite side numbers for any triangle will equal an odd number.***
4. In solution 17, if you add 6,7 and 4 you get 17. In solution 19, if you add 8, 7 and 4 you get 19. For solution 20, you can add 9,1 and 5; for solution 21, you can add 8, 6, and 7; and for solution 23, you can 9, 8 and 6 to get the same sum as the solution. ***So, in each solution, there are always three numbers- one of which is on a different side- , which will always add up to the solution.***

5. The vertices for solution 17 are 1, 2, and 3. The vertices for solution 19 are 4, 1, and 7, for solution 20 they are 5, 3, and 7, for 21 they are 3, 6, and 9 and for solution 23 they are 7, 8, and 9. If you add the vertices for each triangle you get the numbers 6, 12, 15, 18, and 24. These are all multiples of 3. This is very similar to the pattern in the order 3 triangle (but it does not include the multiple 9). This means that for every solution of a magic triangle the vertices should have a sum that is a multiple of 3, and if it is not, it cannot be a solution of the magic triangle. Likewise, the sum of the numbers on the side for solution 17 is 39, for solution 19 is 33, for solution 20 is 30, for solution 21 is 27, and finally for solution 23 is 21. Again, 39, 33, 30, 27 and 21 are all multiples of three. ***So, like in order three triangles, the sum of the vertex numbers of different solutions are always multiples of three. Likewise, the sum of the side numbers of the different solutions are also always multiples of three.***
6. If you take the sum of the vertices (6) in the triangle 17 and add the common difference[3] (11) you get 17. This same rule applies to triangles 19, 20, and 21. But in the case of 23 you must subtract the sum of the vertices and the difference to get 23. ***If you add the sum of the vertices and common difference found earlier will you get the same number as the triangle.***

[3] Each side has an opposite vertex, and this is true for any triangle. For the case of 17, if you add the two side numbers and subtract the opposite vertex you get 11 the same is true for every side and vertex- that is what the common difference means.

Conclusion

The most important thing I learned from this problem is that there are so many possible solutions to one problem. When I started this problem I could barely find any patterns, but after looking closely at the different triangles for many hours I found so many patterns and rules. This was a very fun problem and it taught me a lot about problem solving. I also want to keep working on this to predict solutions for any order triangle. I think I might need some algebraic equations for that. I hope to present those results in the next year's QED.