3-D SUDOKU

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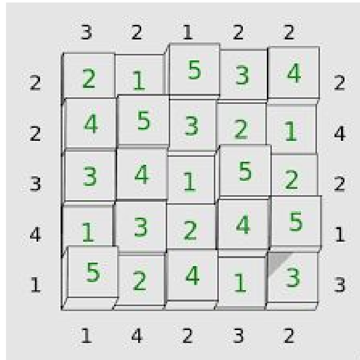
Abstract

What if there was a better version of sudoku? Or if there was a more thought provoking version? In the endless puzzles that relate to mathematics, there is one pattern filled, intriguing puzzle that directly relates to the guidelines of sudoku but in itself is completely different. This puzzle is called Towers, from Simon Tatham's portable puzzle collection. This mind game is most easily thought of as placing towers with different heights on a grid with their locations based on the number of towers that can be seen in each row or column. This project was launched with the goal of finding one or two components of this seemingly simplistic puzzle, but the results that were found drove the project in a completely different way. The dead ends and the successful discoveries led to the project being based upon creating a complete guide on how to solve every four by four tower grid puzzle.

Introduction

The rules of this puzzle are fairly simple- just like sudoku, you are given a square board, n by n, with hints on every row and column. You are supposed to fill it in with towers ranging from 1-n, only using each tower once in that row and column. For example, say you are given a four by four board. There will be a number 1-4 on either ends of all rows and columns.



What do these numbers indicate? They tell you how many towers you should be able to "see" from that point. For example, if you have a row '1-5' like the 2nd row to the bottom in the picture above, they are asking for you to fill in the grid so that you can see 1 tower from one side, and 5 towers from the other. Towers can only be seen if they are taller than the tower in front of it. For example, if you are facing a four story tower, you will not be able to see a three story tower behind it. Conversely, if you had a five story tower behind the four story tower, you would be able to see both towers. You must fulfill the hints of all rows and columns to solve the puzzle successfully.

The history of this problem does not go back very far, but its origins have come from Simon Tatham, a UK based computer programmer who came up of SImon Tatham's Portable Puzzle Collection along with a computer program called PuTTY , which is a "free software implementation of Secure Shell and Telnet for Microsoft Windows and Unix." Towers was created not to copy sudoku, but to further develop many of the ideas that have been brought up in the problem. For example, the idea that there can only be one of the same number in one row and the idea that the highest number in the grid can only be that of the number of "boxes" in a grid or column are both present in Sudoku and Towers.

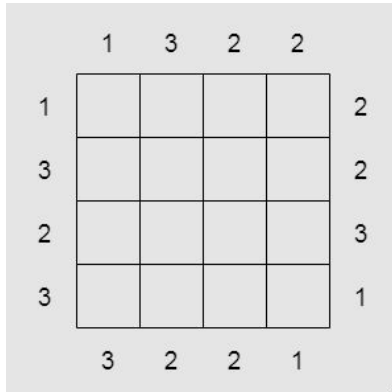
Body

The rules of a standard four by four grid for Towers is that there has to be one of each number in each row or column. So, there has to be one 1, 2, 3, and 4 for each row and each column. You also have to be able to see the number of towers that are "assigned" to each row. For example, if there is a row that is titled 4, there has to be four towers that are seen from that vantage point. To clarify, if someone was standing from that point on the grid, they have to see 4 towers from that certain view. Smaller towers can hide between taller towers.

Here is a list of rules to completely solve a four by four Towers grid- the first section is a list of broad rules, while the second shows scenarios that can apply to different puzzles.

- If you can see a row or column with the hint '1', you put a four down directly in the first space of that specific row or column. Then, if the hint across from that same '1' is a '2', you put a three directly in front of the '2.'
- If a row or column has the hints '1-4', then you can fill out that row or column starting from 1, and then 2, 3, and 4 from the side of the '4.'
- If any of the hints on both sides of a row or column adds to 5, then you know where the four goes. So, if there are two hints '1-4', the four goes directly in front of the one. If there are two hints '2-3', the 4 is placed two away from the hint '2' in that row or column.

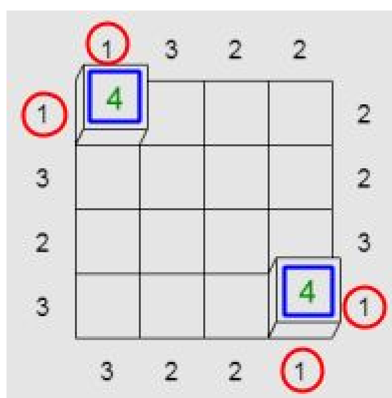
4x4 grid playthrough



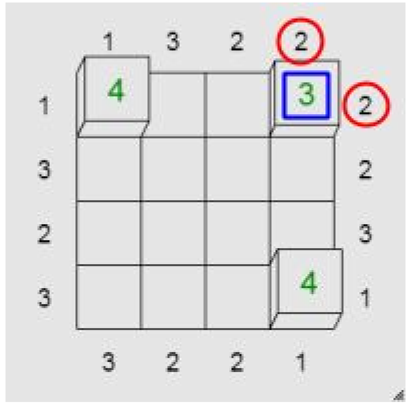
Let’s say that this is the grid you are given to start with.

The key for this diagram:

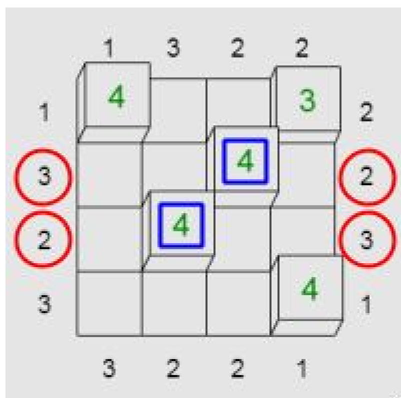
- The circles are around the hints used to figure out the numbers in the blue box.
- The box is around the newly filled in numbers.
- The triangle indicates a number that is filled in using the box as a hint.



We use the rule that all ‘1’ hints have a four next to them, and fill it out.



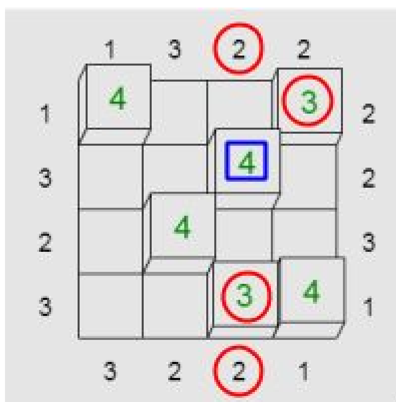
We also know that the combination of the hints '1-2' makes the two end squares 4 and 3, so we know we can fill in a 3.



Using the rule about hints adding up to five (2-3), we fill in the other two fours. This is the most we can fill out using just the rules, so now we move onto the specific scenarios.

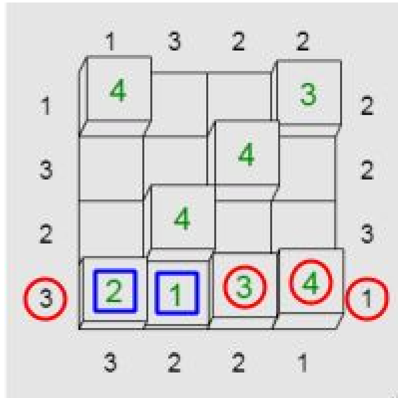
After you've gone through all of the above rules, you can start to look for more specific hints to your puzzle, using what you've solved so far. If you have enough numbers to narrow the options down to one, fill it in- just remember that the options can work both ways (e.g., 1234 is the same thing as 4321, so we didn't include both). If you can't fill out one full row for sure, try to look for overlap, as shown in our playthrough.

- **At any point in time, if you have three numbers in a row or column**, you'll know which number to fill out last, because there can only be one of each number in every row and column. Or, if there are **3 of the same numbers in every single row or column** except one, then you will know where that same number goes the fourth time.
- If there is a column or row that has the hints '1-3', the formations could be (4312), (4132), or (4231).
- If there is a column or row that has the hints '2-3', the formations could be (1432), (2431), or (3421).
- If there is a column or row that has the hints '2-2', then the row or column could be either (1423), (2413), or (3412).
- If there is a column or row that has the hints '1-2', the formations could be (4213) or (4123).

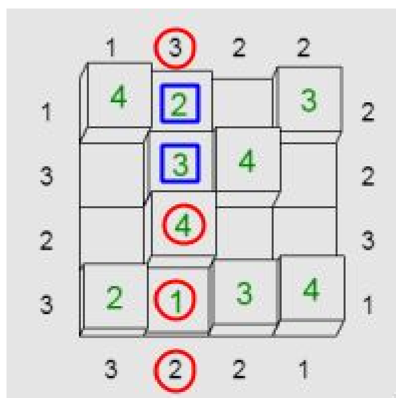


The second column on the right side only works with two scenarios out of the three (1423, 2413, 3412), as the three can't be at the top row- there's already a three there. So we know for sure that there will be a three in the bottom row, next to the four.

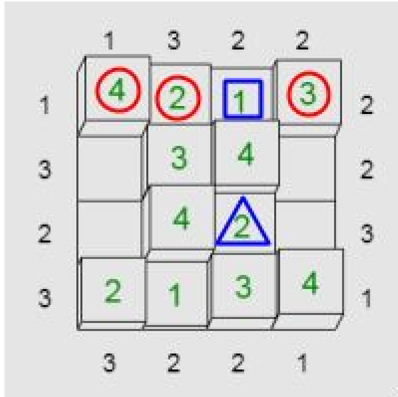
Something to note here: You don't need to fill out a row completely- if you can't, look for overlaps in the possible solutions!



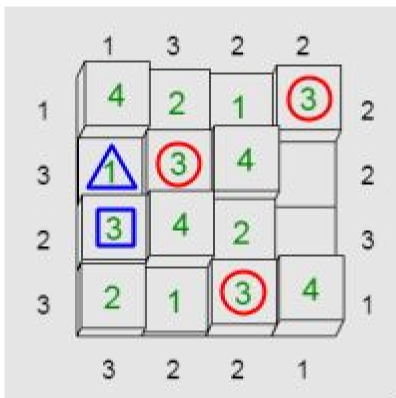
The bottom row has the hint ‘3-1’ with the four and the three next to each other. Out of the three possible scenarios (4312, 4132, 4231) there’s only one option where the three and four are next to each other, so you can go ahead and fill that in.



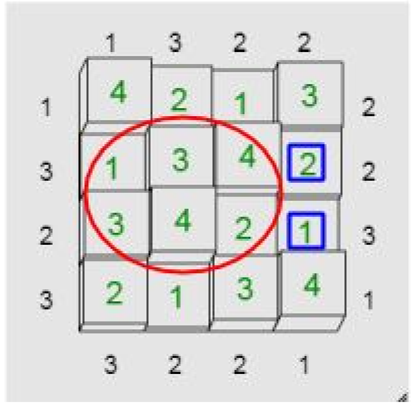
Next, we have the hint ‘3-2’ with a 1 and 4 already filled out next to each other. Out of the three possible scenarios (1432, 2431, 3421), only one fits the requirements of the 1 and 4 being next to each other.



Here, you can refer to the first statement in the scenarios section of the list- you can fill in a one on the top row as that is the only number not in that row, and you can fill out the second column on the right after you do that as the only number missing is a two.



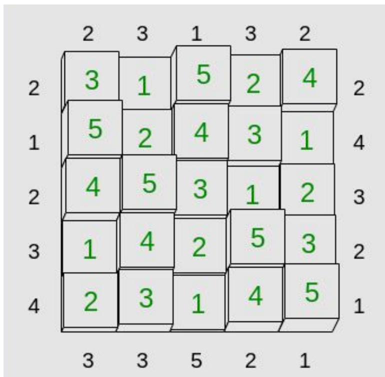
Now, you can see that there are three threes filled out, you know where to fill out the last three. Now, you can fill out the one, as that is the only one left in that column.



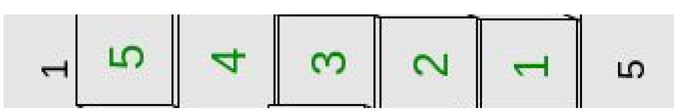
The second and third rows are both missing one number, so you can go ahead and fill that out to complete the puzzle.

While this problem led to many successful discoveries, many dead-ends and difficulties were encountered along the way. For example, after we had solved several Tower grids, we tried to find patterns, whether it was in the arrangement of the number or the arrangement of the "titles" of the rows and columns of the Tower grid. For example, there were no certain numbers that were placed in certain places of the grid because of the placement of some number in a certain place in the grid. That statement, simplified, basically means that there were no extremely distinct patterns in these Tower grid solutions that we had. Another dead end that we faced was also during the pattern-finding process. Instead of trying to find a pattern in the grid, we tried to find a pattern that had to do with the numbers outside of the grid, that is, the hints for each row and column of the Tower grid. This, sadly, also turned out to be a partial dead end. There were some observations that each side of the grid's sum of the hints tended to be a number that was ten through thirteen in a five by five Towers grid, but other than that there was really no complete pattern that we could find. In the end, these dead ends led us to explore the creating of the puzzle itself.

Research before we reached a dead end:

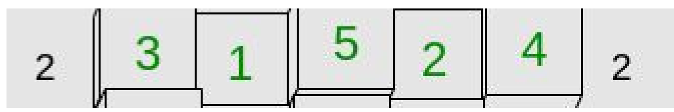


The basic rules of this that there has to be one of each number in each row and each column for the tower grid. For example, if there is a 6 by 6 grid, there has to be a 1, 2, 3, 4, 5, and 6 for each row and column in that specific grid size. Also, there are obvious "correct" (absolutely correct) answers that are hidden in the numbers that are the hints of each column or row. For example, if there is a five by five grid in a puzzle and one of the rows or columns has the hint '1', then you know that the first number relative to the hint has to be five, because no number is "taller" than five. Another example is if the hint on the other side is '2', the number next to that will be '4' as no number is "taller" than 4 other than 5. (Diagram below) Also, if you have a five by five tower grid and a row or column has the hint '5', then you know that that row is going to be ordered 1, 2, 3, 4, and 5, relative to the hint '5'. (Diagram below)



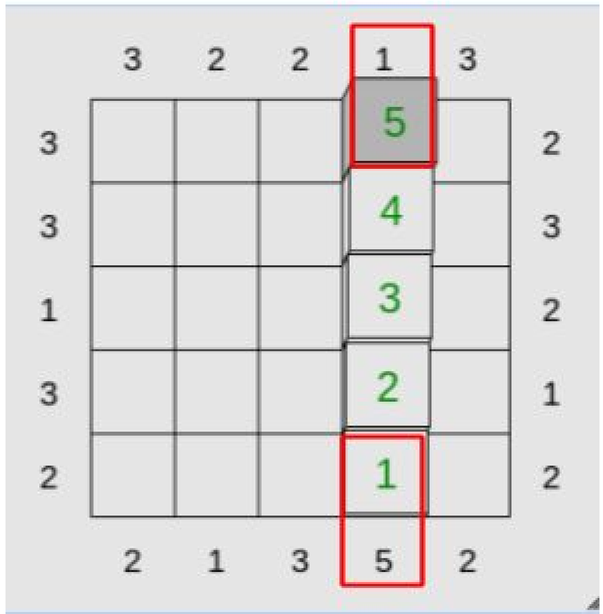
Starting with what is obvious first, we found some strategies such as the two rules described above is the best first step. After doing this, more possibilities and shortcuts will be revealed, because there are less spaces and columns that certain numbers can be input.

Also, if you are working with a 5x5 grid, look for a row or column that has ‘3’ on either side. You know that the middle will be ‘5’ as 5 can be seen anywhere on the grid, meaning you can’t have a five in the second box as it would cover everything behind it. In summarization, you can usually find the location of the 5 by looking at the hints on each side- ‘1-x’ means 5 is all the way to one side, ‘2- 4’ means 5 is one away from the side of the 2, ‘3-3’ means 5 is in the middle. This only works with numbers that add up to six, though- in a ‘2-2’ hint, the five does not have to be one away from a side.

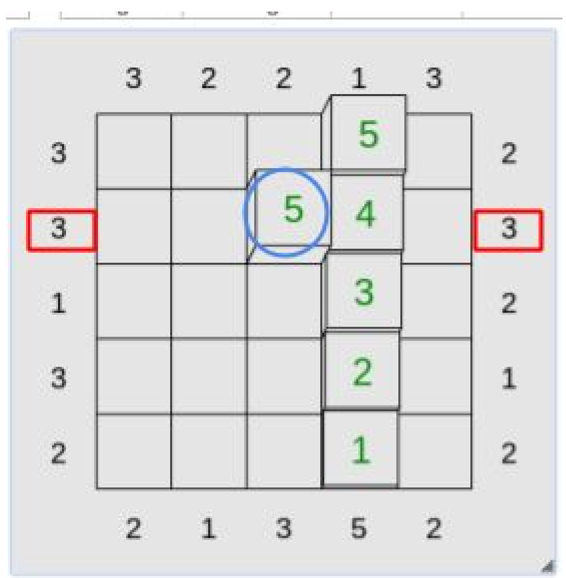


When personally making your own tower grid problem, there are some tips and guidelines to go by. First of all, there doesn’t have to be ANY column or row with the hint ‘5’ in a 5 by 5 grid. Also, not all numbers of the *n* by *n* grid are needed when choosing which numbers to hint the rows and columns. Next, there is no specified limit in the rules that there is a certain amount of numbers that have to be used as hints. For example, the number ‘1’ could be a recurring hint.

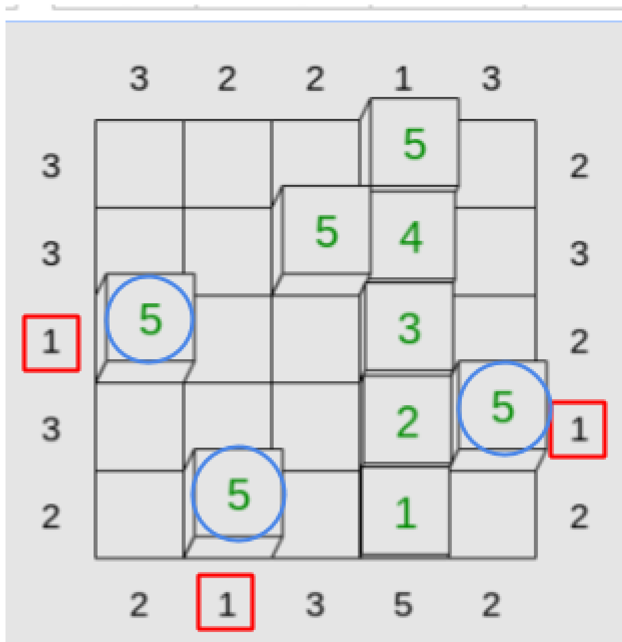
Playthrough of 3D Sudoku?



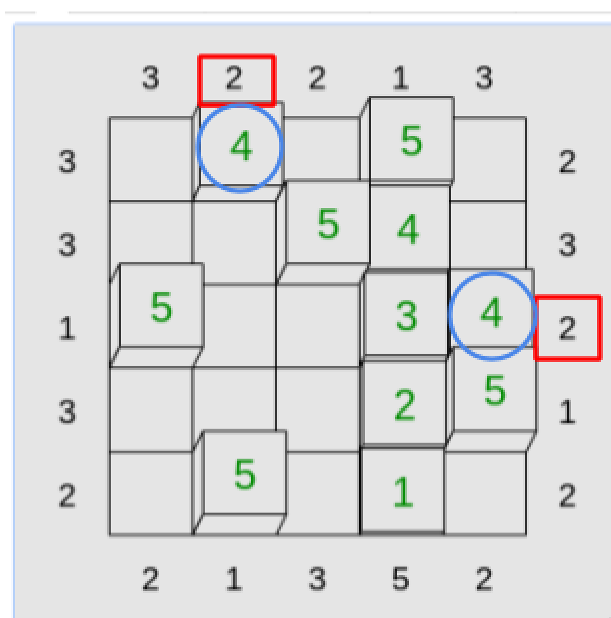
First things first, we checked for a row/column with ‘1-5,’ and filled it in first.



Next, we looked for a ‘3-3’ row/column, and filled in a five in the middle.



Next, we looked for rows/columns with ‘1’ on either side, and filled in the fives. Now, every row and column has a five in it.



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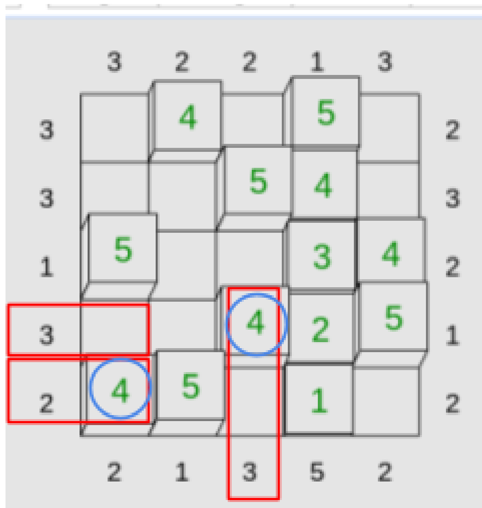
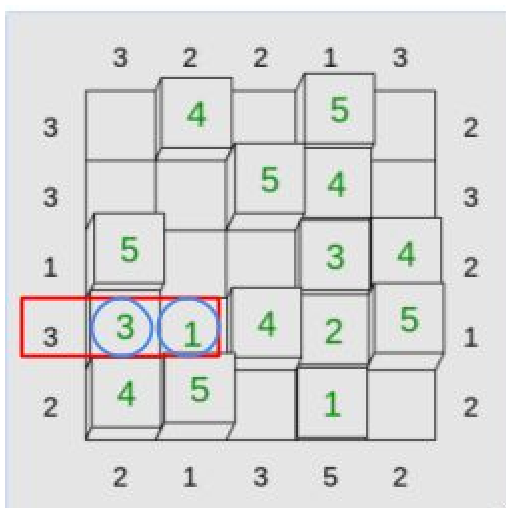
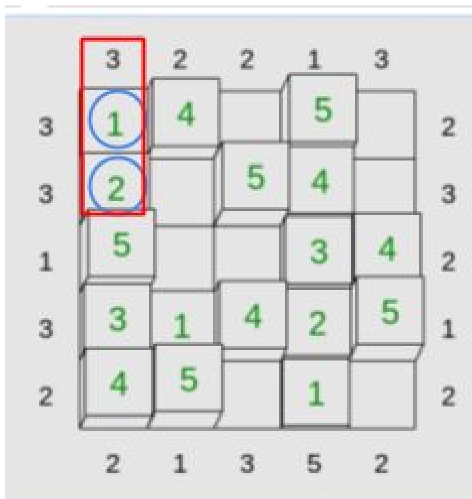
Next, we looked for rows/columns with ‘1-2’, and found two. We filled in the fours.



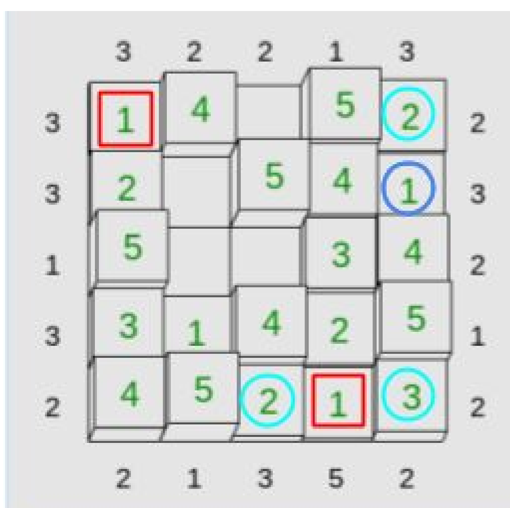
We had two open spots for the remaining fours, and they were the two bottom rows. We knew the 2nd row could not have a four in the first or second box, as you need to see three from the left side, and if it was in the first box, it would only show 2 towers. The second box already had a four in that column, making it not possible. Therefore, we knew that the four in the second row had to be in the third box, leaving only one option for the four in the first row- the first box.



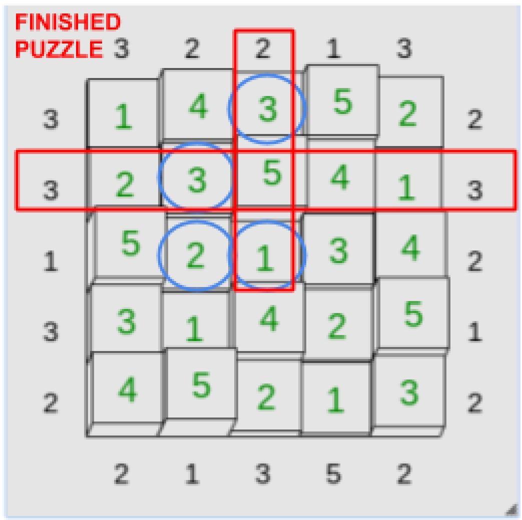
Since the hint requires you to only be able to see three towers, we knew that one couldn't be in the first box, as that would show four towers- therefore, three had to be in the first box, and with elimination, one in the second.



We moved onto the first column to the left, which was only missing a one story tower and a two story tower. Since the hint requires you to see three towers, we knew that the order had to be one and then two, as you can't see any towers behind five- if the order had been two and then one, you would only see two towers.



We know that the 1, circled dark blue, could only go in the second row in the far right column, as the other three open spots had ones in the rows already. After we filled in the one, we knew where the three would go, thanks to the bottom hint, ‘2-2’. If the three was placed behind the two on the bottom row, it would not match the hint. We filled out the two twos by elimination, as those were the only numbers missing from their respective row/column.



Again, by process of elimination, we knew the three had to go in the middle for the top row- it was the only number missing in that row. Also, the three in the second row had to go there for the same reason. Finally, in the second column, there had to be a two and in the third column there had to be a one because those were the only numbers left in those two columns.

From here, we had established a simple guide and away to create your own puzzle, but where was there to go from here? After all, there were way too many possibilities to fully define the steps to completely solving a five by five grid. *“The number of potential Latin squares of size N grows very fast. For a 5×5 square, there are 161,280 possibilities, and for a 10×10, there are over 1047. ”* (Shinn, 2017.) To write a guide for a 5x5 grid would be based more on logic with reasoning more than actual combinations and direct answers. It would take a lot of thinking to create a guide that is not just based on logic, but perhaps, in the future, we can see if there is another more efficient way to approach the 5x5.

Conclusion

Puzzles have always intrigued us, and this one especially captivated our attention. Through developing a guide on how to solve a 4x4 variation of the towers puzzle, we have acquired a deeper understanding on the strategies and inner workings of the puzzle. Instead of only using logic, we have found a way to use math to speed up the process through analyzing the hints. When we were analyzing the hints, we learned to look for patterns that appeared frequently and how to form those into general rules that could be applied to many different situations. Being able to find these overarching rules is a key component in developing problem solving strategies, so it is sublime that we were able to experiment with this project and learn how to find and analyze rules efficiently. While doing our research for this project, which included asking some of the people around us for suggestions and opinions, we came across a new direction- for the 5x5 grid, what if we replaced the middle block with a little man who was looking from the inside out, instead of side to side? To make this problem work, would we have to say what number was missing, or is it still solvable without knowing the mystery number? It is a very intriguing concept, and one that we would love to look into in our free time. Either way, we greatly enjoyed doing a project on such an interesting concept and loved the challenge that came along with it- trying to find problem solving strategies through analyzing patterns.

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