Examining Voter Behavior and Electoral Models: What Should Be Done to Replace the Electoral Model?

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I. ABSTRACT

With many calls to change the way the United States elects its President, I decided to tackle the problem of modeling various forms of election. What makes a good election model, and what is the simplest way to encourage voting and civic duty without making it mandatory, within the constraints of the Constitution? I first start by analyzing the issue at hand, segueing into what makes a voter tick. Once a voter behavior is approximated, then I go into detail on election models, dissecting them and attempting to model them within the context of a United States presidential election.

II. INTRODUCTION

Every election cycle, there’s a renewed call by concerned citizens, media outlets, and politicians seeking an advantage for their party to renovate the Electoral College, the very intricate system in place designed to usher in a new President every four years. However, criticisms have been levied toward the system in recent years: some claim that the system prioritizes large swing states at the expense of every other state, that the will of the people is not accurately reflected, and that the system is the reason third parties such as the Green Party, Libertarian Party, and other don’t exist in the modern American political zeitgeist, among other damning complaints. However, proposals such as the National Popular Vote Bill, various redistricting efforts, and even an entire upheaval of the Electoral College have been entertained, each to varying degrees of success. While there are many potential voting systems and models to replace the Electoral College, what exactly makes a good voting model?

III. VOTING BEHAVIOR

The purpose of this section is to create a model in order to understand how voters will act in any electoral model. In other words, to examine voting models, we must first examine the voters themselves. First, a few simplifying assumptions:

- If a voter is affiliated with a political party, they will vote for that party first and foremost.

American politics, sharply divided as it is today, is rooted in a history of party identification as old
as its two-party system.

- Voters will place an emphasis on current issues. Many elections are determined by a candidate’s position or a party’s history with an issue - a recent example would be the 2008 election, largely seen as a referendum on the policies set by the Bush administration. This point applies largely to independent voters, many of whom don’t typically vote along party lines. They are large enough as a population to warrant their inclusion, and their importance in each election cycle is exemplified by the importance each political party places by catering to them.

- Voters will not necessarily vote. Voter apathy exists, and voters must overcome their apathy in order to cast a ballot.

With these assumptions, a model can be made to predict how voters will vote at any given point, where $i$ is any current issue related to a candidate’s standing on the issue and $p_w$ is a weight, from 0 to 1, signifying the importance of the issue to the voter, all of which are summed together; $c$ is the voter’s perception of a candidate, largely based on media advertising and any emotional perception of the candidate as well during the election; $b$ is the variable for any perceived benefit if the candidate were to be elected; $s$ represents any social ideology, based on race, culture, and demographic, and lastly, $v(t)$ represents this raw score for a candidate’s “votability”, how likely it is a voter will choose a candidate to vote for:

$$
\sum_{i=1}^{n} (p_w * i) + c + b + s = v(t) \quad (1)
$$

All of these variables are organized based on perceived importance in descending order - issues are often polarizing and divisive, a testament to their perceived importance in American politics. Character comes a close second, as a candidate’s values matter to a voter. For example, Jimmy Carter lost the 1980 election on his concessions in the Iran hostage crisis; voters saw this as a weakness, something that Ronald Reagan capitalized on. In addition, candidates advertise themselves as people first and foremost: Reagan was seen as a folksy, friendly character; “you could have a drink” with the younger Bush, circa
2000; Obama was the “cool” candidate who could relate to college students and to the younger generation, all prime examples of selling the candidate. Benefits from any candidate stem from support on certain issues, and as such the two variables are related. Lastly, demographics are intended to cover for a history of support: African-Americans support Democrats by a wide margin as well as Hispanics, while a majority of Caucasians poll with Republicans. In addition, the oft-mentioned liberal/conservative identification split is considered a key demographic that is taken into consideration as well.

What does this model portend? For partisan voters, issues are the deciding factor - people who define themselves as Democrats will tend to support Democratic ideals, and the same goes for self-identifying Republicans, to the point where other variables are overshadowed. As for independents, the full breadth of variables is typically examined before a candidate is picked, given that they will vote, as assumed. Overall, this model is meant to represent the interest of a voter in voting. If \( v(t) \) is low to nonexistent for all candidates, then we can say that the voter has little to no interest in voting. However, if \( v(t) \) is high for at least one candidate, then we can say that the voter has an interest in voting in this election. How will a voter determine which candidate most interests him or her? Simple - compare \( v(t) \) for each candidate. If a specific candidate \( t_0 \) outscores all other candidates running, then the voter will vote for candidate \( t_0 \).

\[
v(t_0) > v(t) \forall \text{ other candidates } t \quad (2)
\]

However, we must account for voter apathy as well. Many pundits, newscasters, and government officials are often stumped by the low turnout of registered voters to the polls. Voter apathy, the phenomenon of perceived apathy among eligible voters, must be overcome before someone decides to vote. Once again, we can attempt to model how voter apathy works, assuming that \( e \) represents socioeconomic standards - historically, upper class, more educated voters tend to vote with more reliability than lower class, less educated citizens; \( s \) represents social demographics, similar to the previous model: Caucasians tend to vote more frequently than minorities, for example; \( a \) represents age, in order to
account for the vast turnout of people older than 50, while younger voters, unless mobilized, tend to vote least frequently. $d$ represents distance - the farther a voter is from a voting booth, the greater the hindrance to voting. $m$ represents mobilization of voters, either via robo-calls, visits, or otherwise. In addition, $m$ includes any perceived threat to their candidate - if a candidate is perceived to be faring well in pre-vote polls, then voters will believe that their candidate does not need their support or that their vote won’t mean as much, in extreme cases where the candidate has a great majority in terms of support. In the other direction, if a voter feels threatened in such a way that their supported candidate may not win, the voter may come out in a show of support, modifying apathy as well; $v_a$ is, once again, a raw score signifying a voter’s given apathy:

$$\pm e - s + a + d \pm m = v_a$$ (3)

We can relate model (3) to model (1) in such a manner:

If $v(t) > v_a$, then a voter will vote in the election. If $v(t) < v_a$, then a voter will stay at home, in effect reducing voter turnout. (Voter Turnout Model, 4)

This lets us predict whether or not a voter will turn out and cast his or her ballot.

**IV. THE ELECTORAL COLLEGE: TWO CANDIDATES**

Now with a model for voting behaviors, what of the actual election? Before the election is determined, it’s best to explain what the Electoral College is comprised of. Simply put, the Electoral College consists of electors, who are voted into their position in a myriad of ways, determined by the state they represent. Every state has electors, determined by the amount of representatives and senators that represent the state in Congress. In addition, the District of Columbia is allotted electors as well, equal to the amount it would have if it were a state, but the number of electors DC can have is capped by the state with the least electors. An absolute majority, $50% + 1$ electors from all electors, is required to become
President. In case of any ties, the election is determined by the House, by the same absolute majority rule. Other restrictions are based on each individual state’s rules. The United States works off of a First Past the Post system of election, where any candidate who collects the greatest amount of votes in a single round of voting wins. Combined with the Electoral College’s criticisms of unfair representation, both systems have been called out increasingly often in the past years.

With these rules and criticisms in mind, here are a few more assumptions, this time for our voting systems:

- No conditions will prevent the voter from voting nor will anything hinder or cause issues with a model. Any sort of fraud, voter discrimination, and other obstacles will not hinder the voter, nor will they hinder the voting count.

- Voters will act rationally and vote according to what they prefer, as per the above model. While other voter behaviors exist, for the purposes of systems where it’s possible to game an election, it’s assumed that voters will vote for their preferences honestly.

- Electors, when using any system in conjunction with the Electoral College, will act faithfully and vote for the candidate they are slated to vote for. While faithless electors are rare (only 27 instances within all presidential elections, none of which have changed the outcome of an election), there’s no other way to account for a faithless elector beyond removing them from the calculations.

- For now, the only parties that voters can vote for are the Republicans and the Democrats, the two mainstream parties of America. Collectively, the Green, Libertarian, and Constitution Parties, the largest third parties, claim only one million voters combined, a sliver of the roughly 240 million people of voting age. The Democratic and Republican parties dominate the two-party system of the United States, so for now, we will assume that their candidates are the only viable candidates. In addition, we will only consider two-candidate elections for the meantime. As a direct
consequence of this assumption, where applicable, votes are cast in an “either-or” manner, with three options: voting for a Republican candidate, voting for a Democratic candidate, or not voting at all.

So what do we desire in a voting system? There are five vital conditions that make a voting system ideal:

- A voting system must have some way to produce a winner: either it must not produce ties or it must have some way to break them.
- A voting system must treat all candidates equally: preferably, if a candidate wins the election, for every possible match-up between that candidate and another on the ballot, he or she will beat the opponent. The candidate that is able to win all other matchups is called the Condorcet winner. Either this must be satisfied or there must be no Condorcet winner. This is known as the Condorcet Winner Criterion (CWC).
- A voting system must treat all votes equally; specifically, there must be no way for a candidate to go from a win to a loss and the alternative to go from a loss to a win WITHOUT having all relevant voters reverse their preferences for the winning candidate A and the losing candidate B. This is known as the Independence of Irrelevant Alternatives (IIA).
- If all voters prefer one candidate C over another candidate D, then candidate D should not be a winner of the election. This is known as the Pareto condition.
- Lastly, if the only change involves a voter shifting a winning candidate E higher up on their preference list, then this should not change the result of the condition. This is known as monotonicity.

Many of the proposed ideal conditions are included in order to simplify an election, our overall goal in creating a better election model. While these conditions seem obvious in retrospect, many election
models, once you include more than two candidates, tend to go awry with such conditions.

A. REDISTRICTING

Many solutions currently exist for reforming the Electoral College without actually changing the spirit of the system in place for over 200 years. One of many solutions that exists involves redistricting the Electoral College districts in order to match that of the popular vote, in order to lessen the wide gap in influence between the populous states and the not-so-populous states. Many, many proposed redistricting plans exist, but one of the more interesting plans, by artist Neil Freeman, proposes that the United States be divided up into 50 states of equal population, in as compact a form as possible in order to avoid possible gerrymandering (the process of manipulating voting boundaries in order to favor one party or candidate over others). The proposal also includes the same amount of electors per state, which would require bumping the current amount of electors up to a number evenly divisible by 50. Since electors and population are standardized to be 6,174,911 people, within a margin of ± 0.02%, modeling such a scenario is simple, and given by the following equation, assuming 9 electors per state:

For any given state \( S \), the electors will be granted as a unit \( u \), where each unit is equal to 9 electors. Where \( c(t) \) signifies all votes cast for a specific candidate \( t \), \( f(t) \) signifies the percentage of the vote received by a candidate \( t \), and \( c_{\text{total}} \) signifies all votes cast in a given state for either candidate, in order for a candidate to receive a unit,

\[
f(t) = \frac{c(t)}{c_{\text{total}}} \geq \frac{c_{\text{unit}} + 1}{2}. \quad \text{(Majority Vote, 5)}
\]

\( c_{\text{total}} \) is represented by \( \sum_{0}^{n} c(t_n) \), the sum of all votes cast for all candidates. \( \text{(Total Vote Count, 6)} \)

If a candidate \( t_0 \) does not satisfy this relation for \( c(t) \), then they do not receive the unit for the state \( S \). 50% ± 1 units, or at least 26 electoral units, are required for a candidate to reach the presidency. Since only two candidates participate in this election, if one candidate does not receive a unit, then the other must receive it, either by breaking a tie or simply by the Majority-Minority Rule that indicates that, in
a two person election, if a candidate A does not receive a majority, then candidate B must have the majority (Appendix A-2).

Advantages to this system include a standard population for any electoral district, making all units equal and interchangeable. By extension, no one unit has a greater influence over another in terms of electoral votes or influence per population. Disadvantages include redistricting every four years in order to keep each district equal and thus interchangeable. The map for this system can be found in Appendix A-1.

**B. FIRST PAST THE POST-POPULAR VOTE**

Taking this previous model and removing the electors from the picture leaves, at its essence, a model for the election of the president via a national popular vote. Many groups have pushed for such a model, with compacts such as the National Popular Vote Bill attempting to create such a model.

A model based off of the popular vote is, in essence, a model where the majority picks the candidate. In order for a candidate to win the election, he or she must collect a majority of the popular vote:

For a candidate \( t_0 \) to win, his or her votes collected, modeled by the function \( c(x) \), the Majority Rule Model, where the domain includes all candidates, must satisfy \( c(t_0) > c(t) \ \forall \text{alternatives} \ t \). (First Past the Post Rule, 7)

This rule is very similar to the Majority Vote model posited above, with the major exception that all a given candidate must do to win is satisfy First Past the Post rules and obtain more votes than every other alternative. With only two candidates, however, the only way to satisfy such a system is for our candidate \( t_0 \) to receive an absolute majority of the votes. This system satisfies all of the conditions mentioned above, and by **May’s Theorem** (Dougherty, May’s Theorem), a model based on Majority Rule is the ideal model for any election involving only two candidates.

**V. ELECTORAL MODELS: THREE OR MORE CANDIDATES**
With an ideal model established for two candidates, now we can explore elections with more than two candidates. In doing so, all prior assumptions and ideal conditions apply with the exception of the Two-Party assumption; its exception is mentioned below.

- With our new condition, now ANY party can field a candidate for the election to the presidency.

Each party nominates one and only one candidate for the election.

Now we can explore a few models that attempt to create the best voting system for the presidential election. To reiterate, the goal is to create a simple model that best follows the ideal conditions stated above.

**A. DIRECT POPULAR VOTE: PLURALITY VOTE**

What if we applied the previous, “ideal” two-candidate model to our new, multiple-candidate election? The rule is the same as before: following First Past the Post rules, a candidate must have the highest vote count in order to win the election. This runs into a few issues once a third candidate is added, however. Consider three candidates, Amy, Brad, and Carl. Their votes are published in the table below:

<table>
<thead>
<tr>
<th>Candidates</th>
<th>Vote Count (out of 60) / Second Vote Count (out of 60)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Amy</td>
<td>23 / 28</td>
</tr>
<tr>
<td>Brad</td>
<td>26 / 26</td>
</tr>
<tr>
<td>Carl</td>
<td>11 / 6</td>
</tr>
</tbody>
</table>

With our rule, Brad would have won the election. However, propose that five of Carl’s supporters, in an attempt to game the election and stop their rival Brad from winning, instead vote for Amy. This violates the IIA Principle: Brad has lost the election now but none of his supporters have betrayed him for Amy. In addition, if we assume that all of Carl’s supporters are ideologically closer to Amy than Brad and would therefore vote for Amy over Brad, then we fail to satisfy the CWC Principle as well: while Amy lost the initial plurality vote, she could have easily beaten Brad had we solely included the two candidates.
in this election.

Here we run across two problems: the possibility that voters can accidentally split a majority and that not all conditions will necessarily be satisfied. Our first problem, vote splitting, can be seen in the 2000 US presidential election between Al Gore, George H. W. Bush, Pat Buchanan, and Ralph Nader. In the end, the margin of victory over Florida, the decisive vote in the election, came as low as a few hundred votes out of millions. Ralph Nader, who had a sizable impact as far as popular vote count goes, was considered a spoiler for Al Gore: since Ralph Nader’s voters would have voted for Al Gore, a more viable candidate who would have won with as little as a few hundred more votes from Nader’s supporters. As for the second problem, we must prioritize what conditions we want in order to pick a “best” system.

B. INSTANT RUNOFF VOTING

So plurality voting doesn’t quite work out in our new world of multiple candidates. What if we were to allow voters to express their preferences, ranking choices instead of simply picking one? This form of voting is called Instant Runoff Voting, which allows voters to rank their choices from 1 to \(n\), where \(n\) is the total amount of candidates running for election. While most Instant Runoff elections require voters to rank all choices or risk invalidating the ballot, in order to agree with the Constitution’s requirement that no voter be prevented from voting in accord with their beliefs, our form of instant runoff voting will only require a first preference. Any other preference will be considered in case the runoff occurs, if no candidate achieves a majority in the first round. If this doesn’t occur, then candidates with the least vote count are eliminated and any ballot preferences are distributed to other candidates in order until a candidate achieves the required absolute majority. Let’s use the 2000 presidential election votes as an example, assuming that 50% of Nader voters preferred Gore second and 20% of voters preferred Bush second:

<table>
<thead>
<tr>
<th>Candidate</th>
<th>Vote Count / Revised Vote Count</th>
</tr>
</thead>
<tbody>
<tr>
<td>Candidate</td>
<td></td>
</tr>
</tbody>
</table>


There is no majority in the first round, so the candidate with the least amount of votes, Ralph Nader, is eliminated. Next, his ballots are distributed by preference, with the remaining 30% of votes considered exhausted, with no more preferences to distribute. These exhausted ballots are discarded, and since Al Gore has a majority, he is considered the winner. Generally, candidates are eliminated one by one but in order to streamline the process, any candidate can be eliminated if, for said candidate $t_0$ and candidate votes determined by $c_{\text{total}}$, given that we start with candidate $t$ with smallest $c(t)$:

If you order the candidates $t_0, t_1, ... t_n$ in a set X by smallest $c(t)$ score to largest, then starting with the automatic elimination of $t_0$, a candidate $t_m$, $0 < m < n$, is eliminated if the relation $\sum_{0}^{m} c(t_m) < c(t_n)$ is true.

(Instant Runoff Voting Elimination Rule, 8)

The direct consequence of this model follows:

If $\sum_{0}^{m} c(t_m) \geq c(t_n)$ holds, then the candidate continues on to the next round of the runoff election. (Runoff Voting Advancement, 9)

In total, there can be up to $n - 1$ rounds in an election with $n$ candidates, although the odds that the race is close enough to ensure that all possible rounds pass even with constant distribution of preference votes and a population as large as the entire United States voting population are astronomically low, enough to merit discarding the idea of a tie.

This model does not come without its own flaws, however. Instant runoff voting does not satisfy monotonicity, the CWC, or the IIA Principle, leaving a few serious questions. Not satisfying monotonicity would seem counterintuitive to a voter: how would raising a candidate cause that candidate to lose? IIA
leads back to the spoiler effect, as seen above with plurality voting. Instant Runoff Voting may be an
easy-to-understand model of election, but the results can be very non intuitive, a strike against it as well.

C. PROPORTIONAL ELECTORAL COLLEGE

The last model of election returns to the Electoral College. Electors are preserved, and each state
has their own number of electors, designated as per the census. This time, however, once the state popular
vote is determined, electors are granted proportionally according to the popular vote count for a state. If a
candidate \( t \) gains \( x \) percent of the vote, then the number of electors \( e \) granted to that candidate by that
state \( S \) is determined as follows, with \( F(t, S) \) with integer outputs designed as the function to collect
electors:

\[
F(t, S) = x \cdot E; \text{ if } x \cdot E - \lfloor x \cdot E \rfloor > .5, \text{ round up; else, round down.}
\]

In case of an elector not being
distributed, the toss-up elector is distributed to the candidate with the greatest percentage of the popular
vote. (Proportional Elector Delegation, 10)

Once all electors have been distributed, if a majority of electors caucus with a candidate \( t \), then
the candidate wins the election. If not, then the brokering process begins. Similar to the IRV elimination
process (Model 8), any candidates with electors who cannot beat the front runner with the sum of smaller
parties’ electors have their electors unpledged, or free from their obligation to vote for the third party.
However, during this brokered convention, third party delegates are brokered as a group. Once a
candidate takes an absolute majority, they win the election.

This method gives third parties some pull, in case either the Republican or Democratic candidates
are not strong enough to take an absolute majority. In that case, brokering is meant to give the third parties
a way to compromise with the much larger parties that they cannot hope to beat normally under a plurality
vote. The actual voting system underlying this revised model is still a plurality vote, so the disadvantages
that arise from a plurality vote still apply. The IIA Principle can still be violated, especially considering that
the actual delegates control the voting process, and CWC can be violated as well.
D. DELIBERATIONS

So what conditions should we favor in our new system? American principles of equality favor a system where everyone has a voice. In order to attract more voters and lower the barrier to voting, a simpler voting process with less moving parts to it is more favorable. That leaves the four principles: since all of the principles are exclusive, proven by Arrow’s impossibility theorem (Arrow, A Difficulty in the Concept of Social Welfare), we must determine the most favorable conditions. The Pareto condition should be favored, and all models mentioned above follow the Pareto condition. In order to simplify votes and avoid voter confusion over preferences and how they affect a candidate, monotonicity should be favored as well. With these conditions, we must choose between the Plurality Vote and the Proportional Electoral College. At this point, we come to a crossroads: do we prefer to keep the historical Electoral College or scrap that entirely for a relatively simple model? The choice is up to the voters.

VI. CONCLUSION

Modeling voter behavior leads to a few hiccups, but for the most part, with the exception of importance, voter behavior is relatively well-defined as well as voter apathy. For two candidates, by May’s theorem, a majority rule election is mathematically determined to be ideal, and by the Majority-Minority rule, with sufficiently large population or an odd number of voters, a winner will always be determined by the first round. For multiple candidates, however, no four ideal conditions will ever be satisfied, which leads to the problem of determining what conditions are important. Extrapolating these models to congressional races, with less voters, and to as far as the local level and multiple-winner races, would be another exercise in and of itself.
VII. WORKS CITED (SEPARATE PAGE)


10. Arrow, Kenneth J. "A Difficulty in the Concept of Social Welfare." *Journal of Political
Neil Freeman’s model (Source 3) assumes that districts will be reapportioned every election year in order to keep equal population in the electoral districts. While he mentions making these districts into states, the divisions within the map are considered within his model above as nothing more than a redistricting of electoral districts; instead of using states as electoral districts, these divisions instead serve their purpose.
A-2: The Majority-Minority Rule

Assume that there are two candidates, candidate A and candidate B, competing for an office via a popular vote. There are a total number of votes $p$ cast in this election, shared between candidates A and B. $p$ is odd in order to prevent a tie between these two candidates. Therefore, since A and B share an odd number of votes, assume that A holds $m$ votes and B holds $p - m$ votes. If $m > \frac{p + 1}{2}$, then candidate A holds a majority of the votes. By construction, $m > p - m$ so therefore B holds a minority of the votes. This relation is true as long as the number of votes cast is odd and there are only two candidates: one candidate will always hold a majority of votes while the other will hold a minority of votes.

The larger the population, the less important it becomes for $p$ to be odd; a greater population will be less likely to force a tie.